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Determination of wood and byproducts elastic moduli using the Impulse Excitation Technique

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TABLE OF CONTENTS

1. Objective

The objective of this white paper is to introduce the theory and methodology for the non-destructive determination of the elastic moduli of wood and byproducts using the Impulse Excitation Technique. It is presented a literature review and the advances made in the application of this technique, already standardized for isotropic materials [1], for the characterization of wood and byproducts.

2. Introduction

Wood, both for its availability and characteristics, was one of the first materials used by man for structural means. It can be classified as a solid, organic compound, formed mainly by cellulose, whose main function is to maintain the support of trees [2]. It is a material that comes from stems that grow annually in diameter and length.

In addition to structural application, wood is also useful as a raw material for byproducts such as glued laminate, laminated plywood, particle boards and wood plastic composites (WPC).

Figure 1 - Bridge over the Montmorency River, Montmorency Forest, Quebec, Canada. It has a span of 44 m, height of 33 m and width of 4.8 m [3].

Due to the microstructural characteristics of its natural origin, the physical properties of wood depend on the direction of loading [2]. The three main directions of the woods are illustrated in Fig. 2 and listed below:

- Longitudinal (*L*): Axis parallel to the fibers (grain direction);
- Radial (*R*): Axis normal to the growth rings (perpendicular to the grain in the radial direction);
- Tangential (*T*): Axis perpendicular to the grain and tangent to the growth rings.
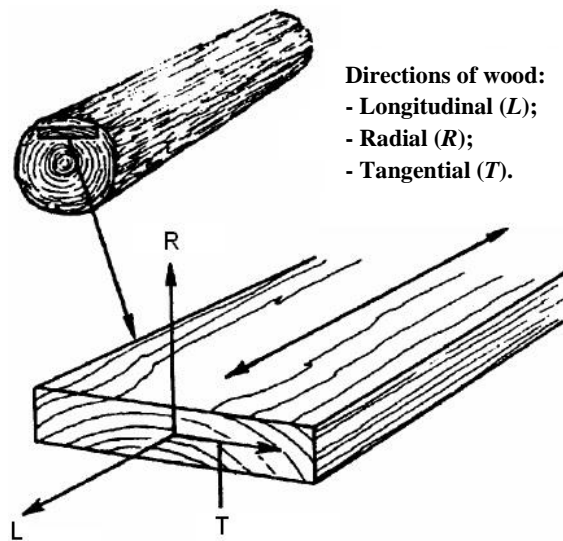
Figure 2 - The three main axes of wood with respect to the grain direction and the growth rings [4].

Wood is classified as an orthotropic material because it presents orthogonal planes of symmetry in which the elastic properties are constant [5]. This classification is an approximation, because depending on the property under analysis it is possible to verify variations, for instance, along the growth rings.

The elastic moduli determination is crucial for the development, selection and quality control of wood and byproducts, as well to simulations and structural calculations. These applications are possible due to the sensitivity of the elastic moduli to the presence of discontinuities, defects, cracks, knots, microstructural changes and chemical composition [6]. One of the non-destructive techniques that has become popular for determining the modulus of elasticity is the Impulse Excitation Technique, the focus of this white paper.

3. Determination of the elastic moduli of wood with the Impulse Excitation Technique

3.1. Technique fundamentals

The Impulse Excitation Technique determines the elastic moduli of a material based on the natural frequencies of vibration of a specimen with regular geometry (bar or cylinder). These frequencies are excited by a short mechanical impulse, followed by the acoustic response acquisition. A mathematical analysis is performed on the acoustic response in order to obtain the frequency spectrum. Then, the dynamic elastic moduli are calculated considering the geometry, mass, specimen dimensions and frequencies obtained by the acoustic response analysis [1].

For the excitation and detection of specific vibration modes, it is necessary to set up specific boundary conditions. Figure 3 presents a specimen support with a manual impulse device and an acoustic sensor positioned to excite and detect the flexural vibration mode.
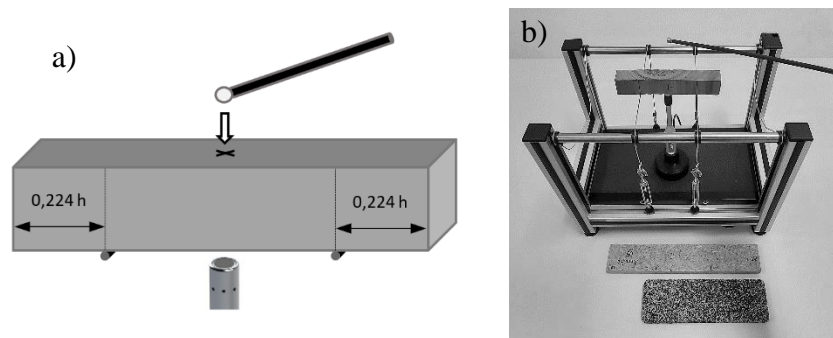
Figure 3 – a) Basic set up to characterize the flexural vibration mode of a bar using the Impulse Excitation Technique [8] and b) SA-BC support for bars and cylinders developed and manufactured by ATCP Physical Engineering.

The Young's modulus obtained by the Impulse Excitation Technique is named dynamic and always greater than or equal to that obtained by a quasi-static test. Nonetheless, it is possible to establish a correlation between them [7].
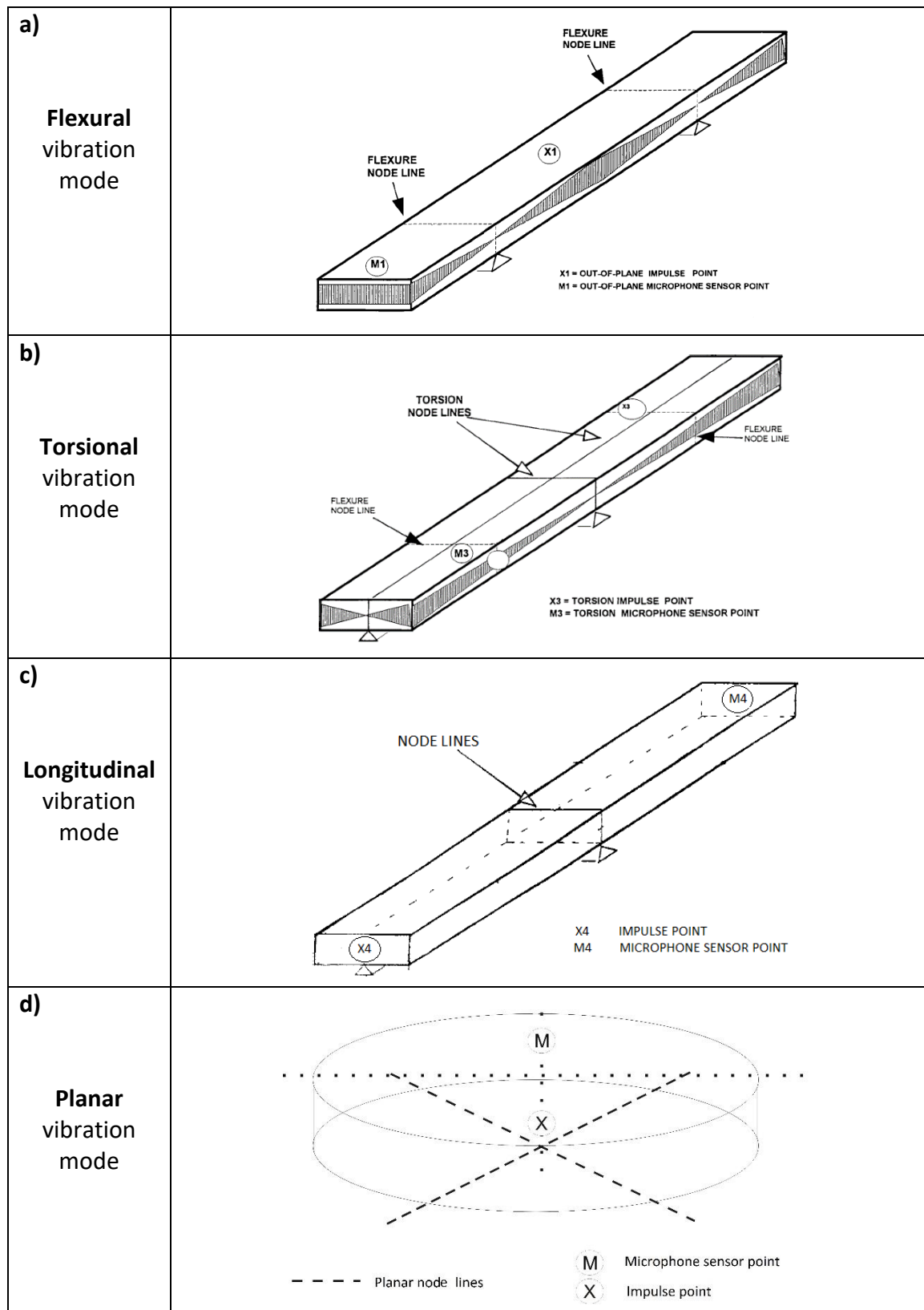
3.2. Vibration modes

A specimen may vibrate in different modes and for each mode there is a specific fundamental frequency. Figure 4 presents the main fundamental vibration modes [9].
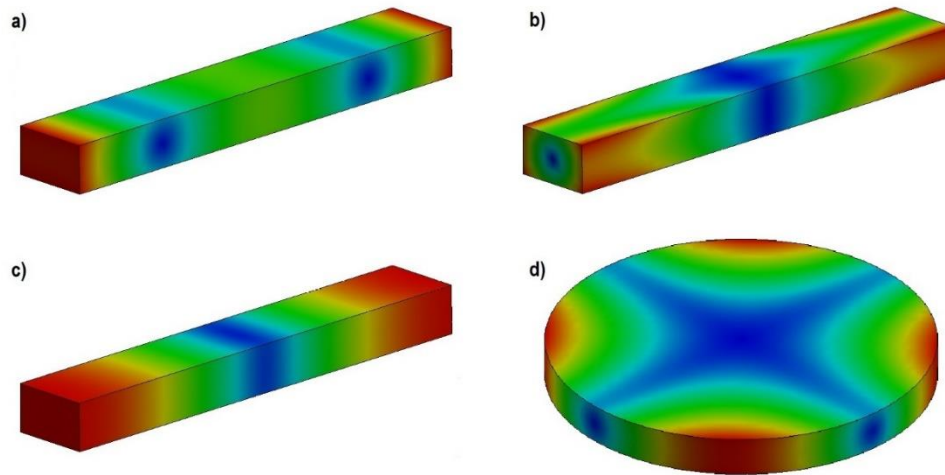
Figure 4 - Fundamental vibration modes: a) flexural, b) torsional, c) longitudinal, and e) planar. The blue region represents the area of minimum amplitude of vibration, whereas the red one represents the area of maximum amplitude.

The boundary conditions imposed on the specimen determine which mode of vibration will be excited.

Figures 5 a-c [1,6] show the optimum boundary conditions for the main vibration modes of a bar, whereas Figure 5d shows the optimum boundary conditions for a disc. The corresponding dynamic elastic moduli are calculated based on the resonant frequencies from the excited vibration mode(s).

a) **Flexural** vibration mode	
b) **Torsional** vibration mode	
c) **Longitudinal** vibration mode	
d) **Planar** vibration mode	

Figure 5 - Boundary conditions for the excitation of (a) flexural, (b) torsional, (c) longitudinal, and (d) planar fundamental vibration modes.

3.3. Elastic moduli of wood

The elastic properties of wood vary according to the orientation of fibers (grain direction) and growth rings. When they are determined using the Impulse Excitation Technique, it is important to take the orientation in consideration to correctly report the results. In the case of wood byproducts, the analysis is similar.

Table 1 - Elastic moduli determined by the Impulse Excitation Technique as a function of the fibers' orientation and vibration modes.
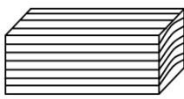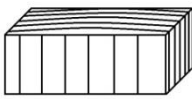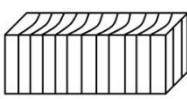
		Fibers' orientation		
		Longitudinal	Tangential	Radial
Vibration mode	Flexural	E_L	E_T	E_R
	Torsional	$G_{eff}\,(G_{LT},\ G_{LR})$	$G_{eff}\,(G_{LT},\ G_{RT})$	$G_{eff}\,(G_{LR},\ G_{RT})$
	Longitudinal	E_L	E_T	E_R

Table 1 presents the elastic moduli of wood that can be determined using the Impulse Excitation Technique in dependence of the orientation of the fibers / grains and of the vibration mode. The terms used in Table 1 are defined as [5,10]:

E_L – Longitudinal Young's modulus;

E_T – Tangential Young's modulus;

E_R – Radial Young's modulus;

G_{eff} – Effective shear modulus determined by the Sonelastic® system, it corresponds to a combination of the G_{ij} moduli shown in parentheses [5];

G_{LT} – Modulus of Rigidity associated to shear strain at the *LT* plane resulting from shear stresses in the *LR* and *RT* planes;

G_{RT} – Modulus of Rigidity associated to shear strain at the *RT* plane resulting from shear stresses in the *LR* and *LT* planes;

G_{LR} – Modulus of Rigidity associated to shear strain at the *LR* plane resulting from shear stresses in the *RT* and *LT* planes.

3.3.1   Young's modulus

- **By longitudinal vibration mode**

When the specimen is excited in longitudinal mode (verify boundary conditions in Figure 5c), the elastic modulus obtained will refer to the orientation or axis parallel to the specimen's length. Therefore, the orientation of the specimen will determine the elastic modulus being measured, as presented in Table 1 (E_L, E_R, E_T or a combination of these directions).

- **By flexural vibration mode**

When a specimen is bended, there are both tension and compression present as shown in Fig. 6. For homogeneous and isotropic materials, the elastic modulus obtained from a bending test coincides with the elastic modulus obtained in an axial test (longitudinal direction). Similarly, the dynamic elastic modulus determined by the flexural vibration mode is the same as by the longitudinal vibration mode [11]. However, in bending, the surface of the specimen is subjected to stresses greater than its center. For this reason, if a specimen presents the stiffness of the surface different from the inside (for example, if there is a stiffness gradient along the thickness), or if the specimen presents small flaws such as pores, cracks and micro-cracks on the surface, there will be a difference between the values determined by flexural and longitudinal vibration modes [10, 12-14].
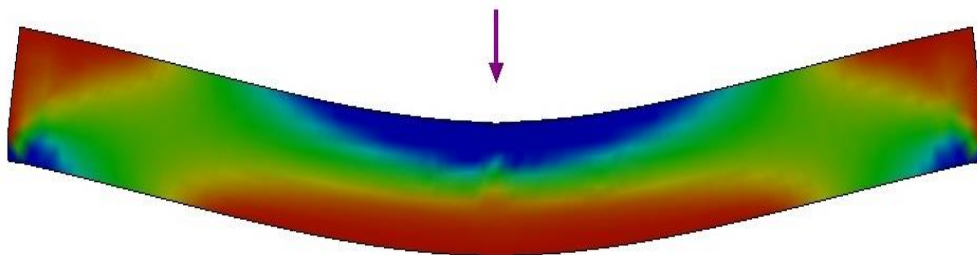
Figure 6 - Regions under tension (red) and under compression (blue) during a bending test.

3.3.2 Shear modulus

- **By torsional vibration mode**

When a specimen is submitted to a torsion test, two values of shear moduli act concomitantly on the material. If there is torsion such as described in Figure 5b, the acting shear modulus are associated with the surfaces being sheared (the four laterals of the specimen). Therefore, the shear modulus calculated using fundamental torsional frequency will correspond to an effective modulus. This way, the result obtained by the Sonelastic® will be a combination of the active shear moduli (Table 1 indicates the active shear moduli that comprise the effective value for each orientation) [5].

3.3.3 Poisson's ratio

The Impulse Excitation Technique is not suitable to characterize the Poisson's ratio of orthotropic materials such as wood. However, based on the Elasticity Theory and using the stiffness matrix, it is possible to obtain correlations between the Poisson's ratio and the elastic moduli. Next, find these relations (stiffness matrix is described in Appendix A):

$$\frac{\nu_{RL}}{\nu_{LR}} = \frac{E_R}{E_L} \, , \ \ \frac{\nu_{TL}}{\nu_{LT}} = \frac{E_T}{E_L} \, , \ \ \frac{\nu_{TR}}{\nu_{RT}} = \frac{E_T}{E_R}$$

To calculate the elastic moduli, it is necessary to estimate the Poisson's ratio. Table 2 presents an estimation and uncertainty according to the average values presented by typical wood. It is worth mentioning that the sensitivity of Young's modulus to the error in Poisson's ratio estimation is low.

Table 2 - Estimated Poisson coefficient and respective uncertainty for the calculation of the elastic modulus of wood in dependence of fiber orientation [5].

Specimen's orientation	Poisson's ratio involved	Poisson's ratio for softwood	Poisson's ratio for hardwood
Longitudinal	ν_{LT} e ν_{LR}	0.40 ±0.05	0.43 ±0.07
Radial	ν_{RT} e ν_{RL}	0.25 ±0.25	0.35 ±0.30
Tangential	ν_{TR} e ν_{TL}	0.20 ±0.15	0.18 ±0.15

3.4. Expected values for softwood

Woods can be classified as low, medium or high-density. Most high-density woods have higher values of elastic modulus and strength [2].

Table 3 presents predicted elastic moduli values as a function of E_L for softwood, showing the elastic properties of this type of wood according to the E_L variation [5].

Table 3 - Predicted elastic parameters as function of E_L for softwood (values in GPa) [5].

E_L	E_R	E_T	G_{LR}	G_{LT}	G_{RT}
6.0	0.6990	0.3667	0.6564	0.6185	0.0518
7.0	0.7710	0.4069	0.6763	0.6366	0.0566
8.0	0.7856	0.4453	0.6962	0.6546	0.0612
9.0	0.8241	0.4821	0.7161	0.6727	0.0655
10.0	0.8601	0.5177	0.7353	0.6907	0.0696
11.0	0.8940	0.5521	0.7558	0.7088	0.0736
12.0	0.9262	0.5855	0.7756	0.7268	0.0774
13.0	0.9567	0.6180	0.7955	0.7449	0.0811
14.0	0.9860	0.6497	0.8154	0.7629	0.0846
15.0	1.0140	0.6806	0.8352	0.7810	0.0881
16.0	1.0409	0.7109	0.8551	0.7990	0.0914
17.0	1.0668	0.7406	0.8750	0.8170	0.0946
18.0	1.0919	0.7698	0.8948	0.8351	0.0979

The Young's modulus obtained by the Impulse Excitation Technique is dynamic and always greater than or equal to that obtained by a quasi-static test. However, it is possible to establish a correlation between the static and the dynamic moduli. Below is an example of correlation, in MPa, between the Young's modulus determined in the longitudinal direction by quasi-static test (E_{LE}) and the dynamic Young's modulus determined in the longitudinal direction by Impulse Excitation Technique (E_{LD}) [7]:

$$E_{LD} = 864.75 + 0.99\,E_{LE}, \text{ for } \textit{Eucalyptus spp} \qquad \text{(A)}$$

$$E_{LD} = 310.15 + 1.07\,E_{LE}, \text{ for } \textit{Pinus oocarpa} \qquad \text{(B)}$$

4. Example – Eucalyptus wood

This example describes the determination of the modulus of elasticity of a Eucalyptus wood (*Eucalyptus spp*) in two orientations using the Impulse Excitation Technique and the Sonelastic® Systems.

## 4.1. Materials and methods

Figure 7 shows a top view of a tree section, the concentric circles correspond to its growth rings. In this figure, three possible specimen cuts are illustrated. The specimens indicated by (1) and (2) present ideal cuts to obtain E_T and E_R, respectively. In (1), the length direction is tangent to the growth rings and in (2), the length direction is parallel to the direction of the growth rings [5].
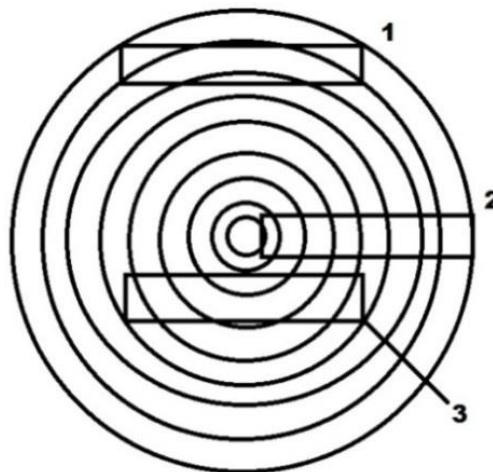
Figure 7 - Identification of the best places to cut the wood specimens to determine E_T (1) and E_R (2). The cut presented in (3) combines both R and T directions.

If the specimen combines the R and T directions as indicated in (3), the values obtained will be effective and result from a combination between E_R and E_T.

Figures 8 and 9 indicate the fibers orientation of the tested wood specimens. The specimen in Figure 8 presents its fibers aligned parallel to its length (these bars were indicated by "L" index). The specimen in Figure 9 has its fibers transversely oriented and there is a combination between the radial and tangential direction along its length (these bars were indicated by "RT" index).
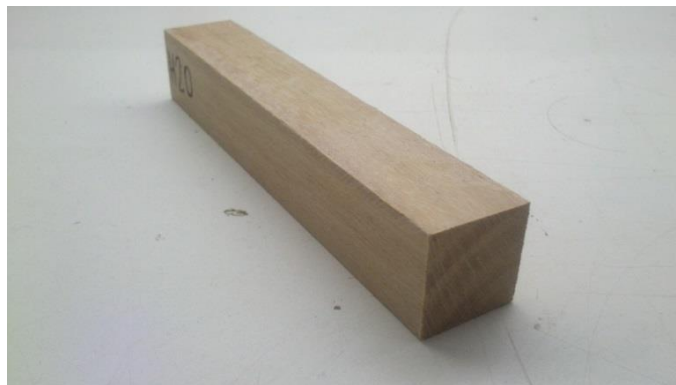
Figure 8 - Specimen with fibers in the length direction (*L*).

Figure 9 - Specimen with fibers in the radial and tangential directions (*RT*).

The dimensions of the specimens were 145 mm x 25 mm x 25 mm and measured by a caliper. The mass was measured by an analytical balance. These parameters are necessary for the elastic moduli calculation.

The specimens were tested with regards the main vibration modes (flexural, torsional and longitudinal) using the SA-BC adjustable support for bars and cylinders, the IED Automatic Electromagnetic Impulse Device, the CA-DP directional acoustic sensor, and the Sonelastic software® (these items are part of the Sonelastic® System for medium-sized specimens shown in Fig. 10).

A Poisson's ratio of 0.25 ±0.25 was adopted to calculate the elastic moduli. As mentioned earlier, this value may vary significantly according to the orientation of the fibers. For this reason, an uncertainty of 0.25 was considered to include all the possible values for this property. Note: the influence of the Poisson's ratio when calculating the elastic moduli is low, this can be verified by checking the uncertainties presented in Tables 4 and 5.

Figure 10 - Sonelastic® equipment developed by ATCP Physical Engineering for elastic moduli determination by Impulse Excitation Technique.

4.2. Results and discussions

The specimens were organized into two groups according to the fiber direction. Tables 4 and 5 show the elastic moduli determined at each vibration mode.

Table 4 - Elastic moduli obtained as a function of the vibration mode ("*RT*" specimens).

Specimen	Longitudinal mode	Flexural mode	Torsional mode
	E_{eff} **(GPa)**	E_{eff} **(GPa)**	G_{eff} **(GPa)**
RT – 01	1.45 ± 0.05	1.50 ± 0.06	1.20 ± 0.01
RT – 02	1.23 ± 0.04	1.30 ± 0.05	1.12 ± 0.01
RT – 03	1.55 ± 0.05	1.67 ± 0.06	1.25 ± 0.01
RT – 04	1.40 ± 0.05	1.45 ± 0.05	1.21 ± 0.01
Average	**1.41**	**1.48**	**1.20**
Standard deviation	0.13	0.15	0.05

Table 5 - Elastic moduli obtained as a function of the vibration mode ("*L*" specimens).

Specimen	Longitudinal mode	Flexural mode	Torsional mode
	E_L **(GPa)**	E_L **(GPa)**	G_{eff} **(GPa)**
L – 01	18.01 ± 0.64	13.16 ± 0.49	1.13 ± 0.01
L – 02	22.39 ± 0.79	17.87 ± 0.66	1.34 ± 0.01
L – 03	21.33 ± 0.76	17.23 ± 0.64	1.43 ± 0.02
L – 04	18.19 ± 0.64	14.19 ± 1.15	1.15 ± 0.01
Average	**19.98**	**15.61**	**1.26**
Standard deviation	2.21	2.29	0.15

The Young's modulus in Table 4 are effective because the specimens present a combination between *R* and *T* directions aligned parallel to their lengths. The values measured for "*L*" specimens are represented by E_L because the fibers are parallel to the length (Table 5).

There is a difference between elastic moduli values obtained in longitudinal and flexural directions, mainly for the "*L*" specimens. This difference is due to the presence of surface defects, which influence more the results in the flexural vibration mode; there may also be a gradient of stiffness along the thickness, so that the surface is more or less rigid than its interior, influencing the results. For wood, there is a tendency for the properties obtained with the longitudinal vibration mode to be superior to those obtained in the flexural mode. [10,12-14].

Finally, G_{eff} consists of an effective shear modulus, which is a combination of the moduli related to the planes being sheared on the torsional vibration. For example, the G_{eff} value for the "*L*" specimens is a combination between G_{LR} and G_{LT}. For "*RT*" specimens, this analysis is even more complex because the direction of the specimen's length does not correspond to a defined orientation (*L*, *R* or *T*).

5. Final considerations

Wood and byproducts are anisotropic materials and its elastic moduli depend on the fibers / grains orientation and vibration mode when tested by Impulse Excitation Technique.

From the orientation of the specimen and the boundary conditions for the flexural and longitudinal vibration modes, it is possible to obtain the main elastic moduli of wood: E_L, E_R and E_T. The Impulse Excitation Technique also allows the characterization of effective shear moduli by torsional vibration mode.

6. References

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Appendix A – Elasticity theory applied to wood

Wood is an anisotropic material and its elastic properties vary according to the loading direction. For this reason, the Elasticity Theory, when applied to this type of material, is more complex than when applied to isotropic materials. This appendix presents how the elastic properties of woods are defined.

- **Introduction: Young's modulus – Hooke's law**

Based on a tensile test of an isotropic material (an annealed metal, for example), at the elastic regime, it is possible to correlate stress and strain as described by Eq. 1 (Hooke's law) [16]:

$$\sigma = E \cdot \varepsilon \tag{1}$$

Figure 15 shows a typical stress-strain curve of a quasi-static tensile test from which the main mechanical properties are obtained. The Young's modulus is the slope of the curve when the specimen is under the elastic regime (curve beginning).
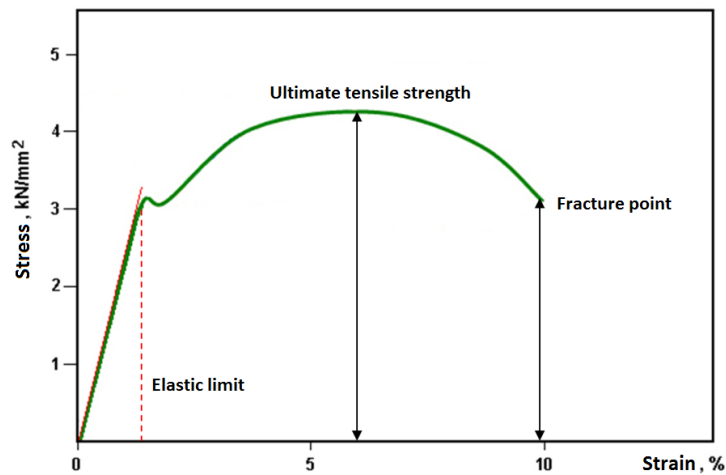
Figure 11 – Stress-strain curve of a high-strength steel [17].

- **Stress and strain – tridimensional model**

The diagram below presents an infinitesimal volume with possible stress that may appear when loading a specimen.
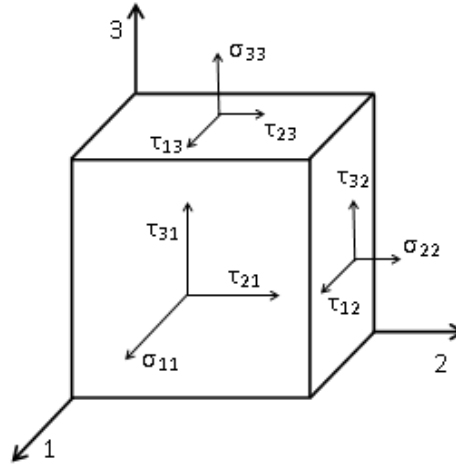
Note that there are two stress types present in this model: the tensile stress represented by σ, and the shear stress, represented by τ. This model may be represented by a 3 x 3 matrix (Equation 2):

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_{33} \end{bmatrix} \tag{2}$$

Assuming that there is a balance of forces within this volume, shear stresses oriented to the same edge of the cube are equivalent (for instance, $\tau_{12} = \tau_{21}$). Therefore, the load applied can be represented by six distinct stresses, as shown in the symmetric matrix below:

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ & \sigma_{22} & \tau_{23} \\ & & \sigma_{33} \end{bmatrix} \tag{3}$$

The same analysis can be made regarding the strain:

$$\left[\varepsilon_{ij}\right] = \begin{bmatrix} \varepsilon_{11} & \frac{1}{2}\gamma_{12} & \frac{1}{2}\gamma_{13} \\ & \varepsilon_{22} & \frac{1}{2}\gamma_{23} \\ & & \varepsilon_{33} \end{bmatrix} \tag{4}$$

- **Anisotropic elasticity**

As it was previously described, wood presents different behavior according to the loading direction, consequently, the stress-strain correlations are not the same for all the directions. Therefore, it is always necessary to consider the orientation of the specimen in order to determine and report its elastic properties.

Generalized Hooke's Law takes into account the stress and strain matrix described in the previous item. It is given by the following expression:

$$\sigma_{ij} = C_{ijkl} \cdot \varepsilon_{kl}, \quad in\ which\ i, j, k, l = 1, 2, 3 \tag{5}$$

Considering the stress and the strain as square matrices of third order, it is possible to conclude that C_{ijkl} is a fourth-order tensor, known as stiffness tensor [11]. Based on the relationships of symmetry described below, it is possible to lower the number of elastic constants from 81 to 21.

$$C_{ijkl} = C_{jikl}, \quad C_{ijkl} = C_{ijlk}, \quad C_{ijkl} = C_{klij} \tag{6}$$

A reduced index notation is used to simplify the correlation between stress, strain and elastic constants, as shown in Table 6.

Table 6 - A four-index notation reduced to a two-index notation [18].

Four-index notation	11	22	33	23	31	12
Two-index notation	1	2	3	4	5	6

Based on previous considerations, the stiffness matrix of an anisotropic material presenting linear-elastic behavior is symmetric and may be described as:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} \tag{7}$$

Another form to represent the stress-strain relation of a material is by using the compliance matrix, as shown in Equation 8:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} \qquad (8)$$

i.e.,

$$[S] = [C]^{-1} \qquad (9)$$

To fully describe a material regarding the elastic properties, it is necessary to find its 21 elastic constants. It is important to highlight that this model represents the elastic properties determined at a specific point in a material, meaning that the described constants may vary from point to point if the material is not homogeneous. Despite the fact that wood is a heterogeneous material, to simplify the model, it is common to consider wood as being homogeneous.

- **Types of material and symmetries**

Despite the properties variation depending on the loading direction, most materials present an internal organization, making possible to simplify the elastic matrix parameters presented in the previous item.

A material that does not present any plane of symmetry is called "anisotropic", and all of its elastic constants are independent [5].

If the material properties vary depending on the loading direction, but has orthogonal planes of symmetry, it is called "orthotropic materials". Wood is generally classified as such. To be able to fully characterize a specimen presenting this type of symmetry, it is necessary to obtain nine independent constants [5].

If a material with an orthogonal symmetry has one of its planes isotropic, it can be classified as being "transversely isotropic" (in the case of wood, this situation would occur if the tangential properties were the same as the radial properties).

Lastly, the simplest possible case that has the smallest number of independent variables is when the material presents isotropy in infinite planes. In that case, the

material is called "isotropic", meaning that the properties do not vary with the loading direction, and the stiffness matrix will depend only on two variables [5].

Anisotropic material (10)

Orthotropic material (11)

Transversely isotropic material (12)

Isotropic material (13)

Legend:
| ■ → Independents |
| 0 → Zero |
| ■—■ → Equals |
| X → $\frac{1}{2}(C_{11} - C_{22})$ |

- **Elastic constants of wood**

According to the previous topic, wood is classified as being an orthotropic material and to fully characterize its elastic behavior, nine independent variables are needed. Next is presented the compliance matrix, described according to its elastic constants. Note that indexes 1, 2 and 3 have been replaced by wood specific indexes: L (longitudinal), R (radial) and T (tangential), respectively [5].

- Compliance matrix:

$$
\begin{bmatrix} \varepsilon_L \\ \varepsilon_R \\ \varepsilon_T \\ \gamma_{RT} \\ \gamma_{LT} \\ \gamma_{LR} \end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_L} & -\frac{v_{RL}}{E_R} & -\frac{v_{TL}}{E_T} & 0 & 0 & 0 \\
-\frac{v_{LR}}{E_L} & \frac{1}{E_R} & -\frac{v_{TR}}{E_T} & 0 & 0 & 0 \\
-\frac{v_{LT}}{E_L} & -\frac{v_{RT}}{E_R} & \frac{1}{E_T} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{RT}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LR}}
\end{bmatrix}
x
\begin{bmatrix} \sigma_L \\ \sigma_R \\ \sigma_T \\ \tau_{RT} \\ \tau_{LT} \\ \tau_{LR} \end{bmatrix}
\qquad (14)
$$

By using this matrix, it is possible to determine the strains as a function of applied stresses if the elements (elastic constants) are all known.