Estimation of the static modulus of elasticity of concrete using the Impulse Excitation Technique

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1. Motivation and goal

The modulus of elasticity is a fundamental property for concrete and it has become increasingly common for designers to specify a minimum value to be met. Currently, however, there is no consensus on the ideal methodology for the characterization or estimation of this property. This context leads to disagreements between structural designers, builders, concrete suppliers and testing laboratories, pointing out to the urgent need of advances in the usual methodologies and standards.

Aiming to promote a more precise alternative to simplify the standardization, this white paper presents a revision of literature and details a methodology for the characterization of the dynamic elastic modulus and the estimation of the static modulus of elastic of concretes based on the Impulse Excitation Technique [1,2].

2. Introduction and rationale

Structural designers usually estimate the static modulus of elasticity of concretes employing models that associate this property with the compressive strength. These are empirical models and they depend on the classification of the concrete under evaluation [3,4]. Despite offering a reasonable estimation for the modulus of elasticity, there are two sources of uncertainty that should be taken into consideration: the first is the validity and the limitations of the employed model [3,4] and the second, the high dispersion of the tests results for the characterization of the compressive strength.

Despite being little explored as a way to estimate the static modulus of elasticity, the Impulse Excitation Technique (IET) [5-7] is a practical and precise alternative to do it. On the contrary of the compressive strength, the dynamic elastic modulus is precisely characterized (typical uncertainty of 1.5% for samples with good superficial finishing) and presents high reproducibility (typically of 0.6%) when employing the IET [1].

Similarly to the compressive strength, there are models that correlate the static modulus of elasticity to the dynamic elastic modulus. However, in these cases, both evaluated properties are only related to the elastic characteristics of the material. The models correlating the static and the dynamic elastic moduli are also empirical and have their own uncertainties and limitations, but the start point is a property measured with high precision, low dispersion, and non-destructively [6,7].
When compared to ultrasonic tests, the Impulse Excitation Technique presents significant advantages for material characterization. For instance, it is not necessary the coupling of transducers to the samples, and the results are less sensitive in what regards the estimation of the Poisson’s ratio [6]. Additionally, the Impulse Excitation Technique allows the simultaneous characterization of the damping.

Concrete is a composite material in which particles or fragments of aggregates are bonded by a matrix [3]. When the aggregates and the matrix are stressed individually both present a linear stress-strain curve allowing the easy determination of the static modulus of elasticity. On the other hand, concrete does not present such linearity because of the presence and appearance of micro-cracks in the interface between both its components [3,4].

The usual classification of concretes is based on their compressive strength. Therefore, concretes exhibiting a compressive strength up to 20 MPa are denominated low-strength concretes; concretes with a compressive strength between 20 MPa and 40 MPa are called medium-strength concretes; and the ones with a compressive strength greater than 40 MPa, high-strength concretes [3]. Figure 1 shows an example of High strength concrete application.

![Figure 1 – Structure made using high strength concrete](image)

Civil construction is the main consumer sector of concrete and contributes significantly to the Brazil GDP. In Brazil, this sector has increased expressively in the 2000s, being possible to verify that by the number of businesses in this sector which has nearly doubled between 2007 and 2012 (Figure 2) [9].
Figure 2 - Number of active businesses in civil construction with one or more employee (Brazil) [9].

The growing number of small and medium-sized companies in this sector demand technological and standardization advances to facilitate and reduce the quality control costs.
3. The modulus of elasticity of concretes

The modulus of elasticity, or Young’s modulus, consists in the proportionality coefficient between stress and strain during the elastic regime of a material. Concrete is a material with nonlinear stress-strain curve, which means that the angular coefficient of the stress-strain curve is not constant. Figure 3 illustrates a typical curve obtained from a compression test of a concrete sample and its main components [3].

![Figure 3 - Typical behavior of the stress-strain curve for concrete and its main components [3].](image)

Because of the non-linearity presented by the stress-strain curve and the testing limitations, different results and interpretations are possible to be made regarding this property. The standards describe distinct methodologies, so there is no general consensus on the best way to define, calculate, predict and test the static modulus of elasticity of concretes [7]. Based on static tests, it is possible to obtain the tangent and secant elastic moduli, both varying according to the points considered of the stress-strain curve (Figure 4). In addition, other aspects such as experimental parameters and the size of the sample may directly affect the determination of this property [3,4].
Figure 4 - Different ways to obtain the modulus of elasticity from the stress-strain curve [10].

The modulus of elasticity of concretes may be also obtained from non-destructive tests, usually based on the natural frequencies of vibration or on the propagation velocity of ultrasonic waves (modulus of elasticity obtained from these tests are classified as dynamic). The employment of non-destructive tests has important advantages, such as the possibility of submitting the same sample to other tests, which turns possible following the evolution of the modulus of elasticity along the curing processes, degradation processes and as a function of humidity and temperature. Therefore, these tests reduce the uncertainty of the results and the total quantity of samples needed for the same study.

Opposed to many mechanical quasi-static tests, dynamic tests submit the material to low stress levels, so there is minimum probability of appearing effects related to the creep and nucleation of micro-cracks on the material. For these reasons, the dynamic modulus is the closest to the initial tangent modulus, which is obtained at the start of the stress-strain curve.

The dynamic modulus is generally 20% greater than the static moduli for high-strength concretes, 30% for medium-strength concretes, and 40% for low-strength concretes [3].
3.1. Parameters affecting the modulus of elasticity of concretes

The elastic properties of concretes largely depend on the properties of their components. Additionally, there are also external factors that may influence the measurements, for example, the humidity of the sample. Figure 5 describes the main parameters affecting the modulus of elasticity of concretes.

- **Aggregate**

  The modulus of elasticity of the aggregate is generally greater than that of the cement paste (matrix), being porosity one of the main variables influencing this property of the concrete aggregates. The more porous an aggregate is, the less stiff it will be, thus, the concrete obtained from these aggregates will present lower stiffness (Figure 6) [3]. For example, it is possible to verify that aggregates of high density (low porosity), such as granite, basalt and volcanic rocks, have a greater modulus of elasticity than the low density aggregates (porous), such as sandstones, limestones and gravels.

  The porosity variation of aggregates affects the modulus of elasticity of the concrete; however, it does not significantly affect the compressive strength, mainly for low- and medium-strength concretes. This fact demonstrates one of the examples in which the influence of a variable is not the same for the mechanical strength and modulus of elasticity, explaining the difficulty in finding a general model to correlate modulus of elasticity to compressive strength [3].

![Figure 5 - Parameters influencing the modulus of elasticity of concretes.](image-url)
The shape, average size, superficial texture and mineralogical composition of aggregates also affect the modulus of elasticity of the concrete. These aspects have an effect on the interface between the aggregate and the matrix, determining the tendency of this region to crack [3,4]. Besides, the volume fraction of the aggregate used in the mixture will also affect directly the modulus of elasticity.

- **Matrix**

As well as the aggregates, the porosity of the cement paste matrix is directly related to its modulus of elasticity, which in turn, will influence the final properties of the concrete. The parameters changing the matrix final porosity are diverse, being the main examples of them the water-cement ratio (Figure 7), the entrapped air content, the mineral additions, and the degree of hydration of the cement.
The hydration of the cement phases during the curing process directly affects the final porosity of the matrix and the main voids found in this region are due to the spaces not occupied by the hydration reaction products and also to the presence of entrapped air.

- **Processing**

Processing a concrete sample involves a mixture of correct quantity of raw material, modeling, curing, extracting from the molds, and the final finishing of the samples (capping and grinding). During the preparation, the curing process must be controlled (Figure 8 illustrates the influence of the curing temperature on the modulus of elasticity) and it must be guaranteed that there is no segregation of the components. Caring for the storage and the transportation of the samples is also important because that the lack of control of these variables may directly compromise the obtained results.

![Figure 8](image)

**Figure 8** - Modulus of elasticity as a function of the age and curing temperature (adapted graph [13]).

- **Testing**

The sample conditions must be evaluated during the test because its properties may be altered according to environmental variables. For example, under humid conditions the modulus of elasticity may present a value around 15% greater than when it is obtained under dry conditions [3].

Besides, for a mechanical quasi-static test, an important variable to be controlled is the loading speed. Intermediate speed must be used because for a very low speed the creep phenomenon of concrete may affect the obtained values and, using a very high loading speed simulates a condition closer to dynamic stress and may overestimate the result [14].
The Brazilian standard ABNT NBR 8522 determines that samples should have 150 mm of diameter and 300 mm of length for the determination of their modulus of elasticity [15]. Nevertheless, there are many standards and distinct procedures that recommend other types of samples (with changes in geometry and also in dimension). It is not recommended the comparison of results obtained from samples with different dimensions and geometries [2-4,7].

3.2. Usual methodologies to estimate the modulus of elasticity

3.2.1 Static test

The Brazilian standard ABNT NBR 8522 describes two methodologies to determine the modulus of elasticity of concretes and suggests the employment of cylindrical samples with 150 mm of diameter and 300 mm of length. The first methodology describes the obtainment of the initial tangent elastic modulus, which, according to the standard definition, is considered to be equivalent to the secant elastic modulus (or chord modulus) between a fixed stress ($\sigma_a = 0.5$ MPa) and 30% of the concrete compressive strength. The second methodology describes the obtainment of the secant elastic modulus, which is defined as being the angular coefficient of the line secant to the stress-strain curve going through two points of the graph. The first corresponding to a stress of 0.5 MPa and the second, to the desired level of stress [15].

For the characterization of the static modulus of elasticity, it is advisable to perform two pre-loading cycles up to a defined stress before effectively registering the stress and strain values for the final calculation. This procedure is adopted so the sample is adapted to the testing machine, avoiding large disturbances at the start of the stress-strain curve (Figure 9).

![Figure 9](image_url) – Pre-loading procedure by ABNT NBR 8522 to determine the initial tangent modulus of elasticity [15].
It is worth highlighting that there is no consensus on the testing parameters that should be employed for the determination of the static modulus of elasticity. National and international standards present significant differences in terms of parameters, which directly affects the final results, such as the applied loading rate, number of pre-loadings that should be performed, maximum stress to which the sample is submitted to, and the type of sample (geometry and dimensions). In addition, it is difficult to determine the strain at the start of the stress-strain curve because of possible imperfections on the samples, the variability of the testing machines and the adaptation process of the top and base to the surface which the load is applied [16]. Inter-laboratorial tests demonstrated that the coefficient of variation of this property may vary between 10% and 15% [6,17]. Therefore, it is recommended to provide information about the adopted testing procedure when reporting the modulus of elasticity of a specific sample of concrete.

3.2.2 Modulus of elasticity prediction based on compressive strength

There are empirical models allowing the estimation of the modulus of elasticity based on the compressive strength. However, these models must be used with reservation and caution because compressive strength and modulus of elasticity are distinct mechanical properties that are differently influenced by the concrete variables [3,4]. There is also no consensus on which model is the best for predicting the modulus of elasticity because there is a lot of questioning to do with the national and international standards applying this type of correlation [6, 18-20]. Figure 10 illustrates the results of the main models used for predicting the modulus of elasticity (Young’s modulus) of concretes based on the compressive strength.

Figure 10 - Available models in standards for predicting the modulus of elasticity through compressive strength value.
The modulus of elasticity is generally specified in project through a correlation with its compressive strength. Based on that, it is not rare for a concrete supplier to be bound to fabricate a material with greater compressive strength than the specified to compensate the uncertainty of the static modulus of elasticity estimation, avoiding the risk of having its material refused [19,21].

The main equations correlating compressive strength to the modulus of elasticity of the concrete according to each standard are:

- **Brazilian Association of Technical Standards – Brazilian Standard 6118 (ABNT NBR 6118) [22]**
  \[
  E_{ci} = \alpha_E \cdot 5600 \cdot \sqrt{f_{ck}}, \quad \text{for concretes with } 20 \text{ MPa} \leq f_{ck} \leq 50 \text{ MPa};
  \]
  \[
  E_{ci} = 21.5 \cdot 10^3 \cdot \alpha_E \left(\frac{f_{ck}}{10} + 1.25\right)^\frac{1}{3}, \quad \text{for concretes with } 55 \text{ MPa} \leq f_{ck} \leq 90 \text{ MPa}.
  \]

- **Fédération internationale du béton - Model Code for Concrete Structures 2010 (fib MODEL CODE 2010) [23]**
  \[
  E_{ci} = 21.5 \cdot 10^3 \cdot \alpha_E \cdot \left(\frac{f_{ck}+6}{10}\right)^\frac{1}{3}
  \]

For both of the models, \(E_{ci}\) refers to the initial tangent elastic modulus at 28 days, \(\alpha_E\) is a dimensionless constant that depends on the type of aggregate used for the concrete fabrication (Table 1) and \(f_{ck}\) is the specific compressive strength of the concrete.

<table>
<thead>
<tr>
<th>Aggregate</th>
<th>(\alpha_E)</th>
</tr>
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<tbody>
<tr>
<td>Basalt and dense limestone</td>
<td>1.2</td>
</tr>
<tr>
<td>Quartz</td>
<td>1.0</td>
</tr>
<tr>
<td>Limestone</td>
<td>0.9</td>
</tr>
<tr>
<td>Sandstone</td>
<td>0.7</td>
</tr>
</tbody>
</table>

- **American Concrete Institute – 318: Building code requirements for structural concrete (ACI 318) [24]**
  \[
  E_c = 0.043 \cdot \omega_c^{1.5} \cdot \sqrt{f_c}, \quad \text{for concretes with } 1440 \text{ kg/m}^3 \leq \omega_c \leq 2560 \text{ kg/m}^3.
  \]
  \[
  E_c = 4732 \cdot \sqrt{f_c}, \quad \text{for concretes with usual specific mass.}
  \]
In which $E_c$ is the secant elastic modulus obtained from the start point up to a level of stress equal to $0.45f_c$, $\omega_c$ is the concrete density (in kg/m$^3$) and $f_c$ is the specified compressive strength.

- European Committee for Standardization. Eurocode 2: Design of Concrete Structures (EUROCODE 2) [25]

$$E_{cm} = 22 \cdot \left(\frac{f_{cm}}{10}\right)^{0.3}$$

In which $E_{cm}$ is the secant elastic modulus obtained from the start point up to a level of stress equal to $0.4f_{cm}$ and $f_{cm}$ is the average compressive strength.
4. Methodology for estimating the static modulus of elasticity of concretes using the Impulse Excitation Technique

4.1. Dynamic tests and Impulse Excitation Technique

A practical and precise alternative way to obtain the static modulus of elasticity of a concrete sample is to do its estimation based on the dynamic elastic modulus [6]. The characterization of the dynamic elastic modulus is especially relevant for applications in which the concrete is submitted to dynamic loads; for example, in applications where sudden loads occurs in the structure [26]. Because very little stresses are applied during the determination of this property, there is no micro-cracking nucleation and neither creeping effect. Thus, it is possible to consider that the dynamic elastic modulus is associated with purely elastic phenomena [4].

The ASTM C215 and ASTM E1876 standards describe the main methodologies for the characterization of the dynamic elastic moduli of concrete samples. These standards predict the characterization of natural vibration frequencies of the samples and, together with mass and dimensions, it is performed the calculation of the dynamic elastic modulus. In the case which the excitation of the sample occurs by an impulse (a tap), this variation is called Impulse Excitation Technique (IET) [1,2]. This technique has gained space in the civil construction sector and is an alternative with great potential to estimate the static modulus of elasticity of concrete [6,7,27-30]. Next, find the main advantages of employing this technique:

— Allows the reduction of the total quantity of samples in one study and intercalates the test with other processes for being a non-destructive test;
— Provides more precise results than static tests because of the lower quantity of variables and less susceptibility to experimental errors [27,30];
— Speeds up the obtainment of results (the characterization takes only few seconds to be carried and it may be repeated as many times as needed);
— Allows following closely the modulus of elasticity of a sample as a function of time (for example, to follow the curing process; Figure 11);
— Turns it possible to evaluate the progressive damage of structural elements since the crack lowers the material’s stiffness and, as a consequence, it reduces the natural frequency of vibration [26];
— Turns it possible to monitor the variation of the modulus of elasticity of the same sample as a function of other variables, such as temperature or the number of thermal cycles (ASTM C666 [31]).

![Graph](Image)

**Figure 11** - Monitoring the dynamic elastic modulus of three samples as a function of their age [32].

### 4.2. Relation between the static and dynamic modulus of elasticity

The difference between the static modulus of elasticity and the dynamic elastic modulus of a concrete sample is mainly due to the fact that this material possesses a viscoelastic behavior, meaning that its behavior varies according to the strain rate applied during the test (Figure 12).

![Graph](Image)

**Figure 12** – Curves obtained from theoretical models demonstrating the influence of the strain rate on the stress-strain curves of a concrete [14].
The lower the rate applied the longer the time for the relaxation of stresses and the lower the slope of the stress-strain curve. On the other hand, the higher the rate, the shorter the time for the relaxation of stresses and the greater the slope of the stress-strain curve (Figure 12). The strain rate applied during a dynamic test is always superior to the one applied at static tests, as well as the strain levels applied at a dynamic test are lower. Therefore, the dynamic elastic modulus will be always greater than or equal to the modulus obtained from a quasi-static test, getting close to the initial tangent elastic modulus of the concrete [14,33,34].

Below find the main equations for predicting the static chord elastic modulus ($E_c$) based on the dynamic elastic modulus ($E_d$). We suggest applying the model proposed by Popovics [35] to predict the static modulus of elasticity because it was tested and presented good results for low- and medium-density concretes at different ages (between one day and one year), and also at different classes of compressive strength (between 5.4 Mpa and 82.7 Mpa) [6,35].

$$E_c = k \cdot E_d^{1.4} \cdot \rho^{-1}$$

In which $k$ is a constant which depends on the units used ($k = 0.107$ when the modulus is expressed in Pa and density in kg/ m³) and $\rho$ is the concrete density.

Note: The static chord elastic modulus described by the Popovics’ model corresponds to the modulus obtained by using the ASTM C469 standard, in which this property is obtained from the slope of the line passing through two specific points of the stress-strain curve: the first corresponds to the level in which the sample presents a strain of 5x10-5; and the second to the level in which 40% of the compressive strength of the concrete is reached. The value of this modulus tends to be slightly lower than the initial tangent elastic modulus ($E_{ct}$) described by methodology of the Brazilian standard ABNT NBR 8522, since that, for the Brazilian standard, the second point to be considered corresponds to a stress level which is 30% of the compressive strength.
4.3. Methodology for predicting the static modulus of elasticity of concrete

Following, it is presented the flow chart for the estimation of the static modulus of elasticity of a concrete sample based on the dynamic elastic modulus obtained through the Impulse Excitation Technique.

![Flow chart of the methodology for the estimation of the static modulus of elasticity based on the dynamic elastic modulus.](image)

**Step 1:**

The first step consists in determining the geometry and dimensions of the sample. The Impulse Excitation Technique (IET) allows the use of rectangular-bar samples, cylinders and discs [1,2]. We suggest the cylindrical geometry with 100 mm of diameter and 200 mm of length, which is adequate for both the characterization by IET and for the determination of the compressive strength.

**Step 2:**

The second step consists in preparing the sample, and assess the mass and dimensions. During the concrete preparation stage, it is necessary to guarantee that there is a correct quantity of mixture of raw materials. Molding and the posterior extraction from the molds must be made in such a way that these processes will not cause flaws on the surface of the sample (pores or cracks). In addition, the curing process must be controlled (humidity and temperature) and it is necessary to guarantee that there is no
segregation of the components. For a greater dimensional precision, it is suggested to grind the surfaces of the sample.

To assess the mass and dimensions, it is necessary to use precise and calibrated tools, in addition the measurements of dimensions must be taken from at least three points along the sample. The greater the precision when obtaining these parameters, the lower the uncertainty of the dynamic modulus of elasticity characterized using the Impulse Excitation Technique.

**Step 3:**

The third step consists in characterizing the dynamic elastic modulus using the Impulse Excitation Technique. It is recommended to use the Sonelastic® equipment (Figure 14) for this purpose and the procedures described in Chapter 5. For details about the characterization of the dynamic modulus of elasticity of concretes using the Impulse Excitation Technique, please, see page 20.

![Image of Sonelastic® equipment](image)

**Figure 14** - Basic configuration of the Sonelastic® equipment used for the characterization of concrete samples.

**Step 4:**

The fourth step consists in applying the Popovics’ model [35] for estimating the static (chord) modulus. Below, find the model correlation (Topic 4.2, page 14):

\[
E_c = 0.107 \cdot E_d^{1.4} \cdot \rho^{-1}
\]

The input values correspond to the density of the concrete (\(\rho\)), in kg/m³, and to the dynamic elastic modulus (\(E_d\)), in Pa. The first parameter may be obtained from the
ratio between the mass and the volume of the sample and the second, from the characterization of the dynamic elastic modulus using the IET (Step 3).

The uncertainty of the obtained value for the chord elastic modulus can be calculated based on the uncertainties of the input parameters \(E_d\) and \(\rho\), through the correlation described next:

\[
\Delta E_c = E_c \cdot \sqrt{\left(1.4 \cdot \frac{\Delta E_d}{E_d}\right)^2 + \left(\frac{\Delta \rho}{\rho}\right)^2}
\]

In which \(\Delta E_c\), \(\Delta E_d\) and \(\Delta \rho\) are the uncertainties related to the chord elastic modulus, dynamic elastic modulus and density, respectively. It is worth highlighting that the uncertainty calculation does not include the uncertainty of the model.

In the case of the density being obtained from the ratio between the mass and the volume of the sample, the uncertainty related to this parameter can be calculated by the following relations:

- For rectangular bars: \(\Delta \rho = \rho \cdot \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta w}{w}\right)^2 + \left(\frac{\Delta t}{t}\right)^2}\)

- For cylinders: \(\Delta \rho = \rho \cdot \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta l}{l}\right)^2 + \left(2 \cdot \frac{\Delta d}{d}\right)^2}\)

In which \(\Delta \rho\), \(\Delta m\), \(\Delta l\), \(\Delta w\), \(\Delta t\) and \(\Delta d\) are uncertainties associated with the obtainment of density, mass, length, width, thickness and diameter, respectively.

**Step 5:**

The fifth and last step consists in making the results report. Besides the methodology and model used, this report must include all data affecting the results (material, processing, cure, sample geometry, testing conditions and employed model).

Find on the next page a report model with the most relevant information for the estimation of the chord elastic modulus of a concrete (Figure 15). An example of the application of this methodology is described in Chapter 6 (Estimation of the static modulus of elasticity of a concrete employing the proposed methodology, page 28).
<table>
<thead>
<tr>
<th>General information</th>
<th>Date:</th>
<th>Time:</th>
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<tr>
<td>Responsible:</td>
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<td>Place:</td>
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</tbody>
</table>

### Material and test information

Name / Part number:

Mix: Water / cement ratio:

Type of aggregate: Cement:

Cure conditions: Age:

Sample geometry:

Dimensions*:

Mass*:

Density*:

*Include the respective uncertainties.

** Calculation of apparent density uncertainty:

\[
\Delta \rho = \rho \left( \frac{\Delta m}{m} \right)^2 + \left( \frac{\Delta \rho}{\rho} \right)^2 \]

### Information on the non-destructive test (Impulse Excitation Technique)

Environ. temp. and humidity:

Standard used:

Vibration mode:

Equipment used:

### Information on the model used to estimate the static modulus of elasticity

Model used *: Popovics ->

\[
E_c = 0.107 \cdot E_d \cdot \rho^{-1}
\]

*We suggest the model proposed by Popovics:

\[
E_c = 0.107 \cdot E_d \cdot \rho^{-1}
\]

The static modulus of elasticity estimated by the Popovics model (Ec) corresponds to the chordable modulus obtained between two points of the stress-strain graph: the first at a deformation level of 5.00E-5 and the second at 40% of the compressive strength.

### Results

Dynamic modulus of elasticity (Ed):

Estimated static modulus of elasticity (Ec)*:

*From the model proposed by Popovics, uncertainty can be obtained by:

\[
\Delta E_c = E_c \cdot \left( \frac{\Delta E_d}{E_d} \right)^2 + \left( \frac{\Delta \rho}{\rho} \right)^2
\]

**Figure 15** – Example of a report that may be used to specify the obtained results using the described methodology.
5. Detailing the characterization of the dynamic elastic moduli of concretes using the Impulse Excitation Technique

5.1. Technique fundamentals

The Impulse Excitation Technique (ASTM E1876 [1] standard) essentially consists in determining the elastic moduli of a material based on the natural vibration frequencies of a sample with regular geometry (bar, cylinder, disc or ring). These frequencies are excited by a brief mechanical impulse (a tap), followed by the acquisition of the acoustic response using a microphone. A mathematical treatment is performed on the acoustic signal in order to obtain the frequency spectrum (Fast Fourier transform). Based on this, the dynamic elastic modulus is calculated using predetermined equations by the standard, which considers the geometry, mass, sample dimensions and frequency obtained by the equipment [1,2].

For the excitation of the desired vibration modes, it is necessary to set up specific boundary conditions. Figure 16 presents an example of a sample holder system, of an excitation position, and of acoustic response acquisition considering the flexural vibration mode of a cylinder.

**Figure 16** – a) Basic set up for the characterization of a bar based on the flexural vibration mode using the Impulse Excitation Technique [36] and b) SA-BC adjustable support for bars and cylinders; part of the Sonelastic® equipment developed and manufactured by ATCP Physical Engineering.
5.2. Vibration modes

A sample may vibrate in different ways and for each mode there is a specific fundamental frequency. Figure 17 presents the main fundamental vibration modes of a cylinder and of a disc.

![Figure 17 - Fundamental vibration modes: a) flexural, b) torsional, c) longitudinal, and d) planar. The blue regions represent the areas of minimum amplitude of vibration, whereas the red ones represent the areas of maximum amplitude.](image_url)

Boundary conditions imposed during characterization are responsible for determining which vibration mode will be excited. The fundamental frequency of these modes depend on the geometry, mass, dimensions and elastic moduli of the material.

Figures 18a-c present the optimum boundary conditions for the main vibration modes of a rectangular bar and of a cylinder, whereas Figure 18d presents the optimum boundary conditions for the planar mode of a disc [1]. The dynamic elastic modulus is calculated by employing the equations described by the ASTM E1876 standard [1], which is based on the resonant frequencies of the sample, its mass and its dimensions.
a) Flexural mode

Rectangular bar:

Cylinder:

b) Torsional mode

Rectangular bar:

Cylinder:
<table>
<thead>
<tr>
<th><strong>c) Longitudinal mode</strong></th>
<th>Rectangular bar:</th>
</tr>
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<tbody>
<tr>
<td></td>
<td><img src="image" alt="Rectangular bar" /></td>
</tr>
<tr>
<td><strong>Cylinder:</strong></td>
<td><img src="image" alt="Cylinder" /></td>
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</tbody>
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<table>
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<tr>
<th><strong>d) Planar mode</strong></th>
<th>Disc:</th>
</tr>
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<td></td>
<td><img src="image" alt="Disc" /></td>
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| Nodal lines | Acoustic sensor point | Excitation point |

**Figure 18** - Boundary conditions set up for the sample for the excitation of (a) flexural, (b) torsional, (c) longitudinal, and (d) planar fundamental vibration modes.
5.3. Elastic properties and vibration modes

The main elastic properties of a material are the modulus of elasticity (or Young’s modulus), the shear modulus and the Poison’s ratio. The Impulse Excitation Technique allows the characterization of these three properties which are detailed next as a function of the applied vibration mode.

5.3.1 Modulus of elasticity (Young’s modulus)

- **Longitudinal vibration mode**

  When the sample is excited in longitudinal mode, the elastic modulus obtained refers to the orientation parallel to the sample’s length. Therefore, this modulus is equivalent to the dynamic value obtained from a tensile or compressive test.

- **Flexural vibration mode**

  When a sample is flexed, both tension and compression stresses occur, as pictured in Figure 9 [37]. For homogeneous and isotropic materials, the elastic modulus obtained from a bending test coincides with the elastic modulus measured longitudinally. Therefore, the value of the dynamic elastic modulus obtained through the flexural vibration mode is the same as the one obtained through the longitudinal vibration mode [37]. Nevertheless, it is known that, when flexed, the surface of the material is the region submitted to the greatest values of tension. For this reason, if the stiffness of sample’s surface is different from that of its center (for example, if there is a stiffness gradient along the thickness), or if the sample presents some flaws such as pores, cracks and micro-cracks on the surface, there will be a difference between the values obtained through flexural and longitudinal vibration modes. There is a range of publications presenting a comparison between the modulus of elasticity obtained from both the flexural and longitudinal vibration modes [5-7].

![Figure 19 - Tensile (red) and compression (blue) regions of stress during a bending test.](image-url)
5.3.2 Shear modulus

- Torsional vibration mode

One of the main ways to obtain the shear modulus from a static test is through a torsion test. In the case of the Impulse Excitation Technique, the principle is similar, but in this case it is necessary to provide the boundary conditions for the sample to vibrate under this mode (see Figure 18b). In the case of the rectangular samples, there is only the need to excite the sample near one of the lateral axis outside de nodal lines and acquire the signal from another symmetrical edge. However, in the case of cylindrical samples, it is necessary to couple small tabs near the edges of the sample so that it is possible to excite and acquire the acoustic response from the torsion. In the appendix of this white paper it is describe how to couple these tabs to the cylindrical samples.

5.3.3 Poisson’s ratio

The characterization of the Poisson’s ratio through the Impulse Excitation Technique occurs indirectly, based on the correlation between the modulus of elasticity and the shear modulus of the material [1], which is given by the following equation:

\[ \nu = \frac{E}{2G} - 1 \]

In which \( E \) is the modulus of elasticity, \( G \) is the shear modulus and \( \nu \) is the Poisson’s ratio.

5.4. Differences between the ASTM E1876 and ASTM C215 standards

There are many ASTM standards focused on the characterization of elastic moduli using the Impulse Excitation Technique [1]. The main distinction between them is related to the specifics of the material to be characterized. For example, the ASTM C1259 standard describes the application of the technique for advanced ceramics, whereas the ASTM C1548 standards, for refractory ceramics. The standard describing the characterization of concrete sample is ASTM C215, whilst ASTM E1876 describes the technique to be used in general, disregarding the material specifics.

Regardless of the standard used, they all possess the same fundamentals, which are based on studies developed at the beginning and halfway through the last century [38,39]. Despite that, however, it is possible to verify the existence of some differences...
between these standards, mainly between ASTM E1876 and ASTM C215. Next, find the main diverging points between these standards:

— Sensor used for acquiring the signal:

ASTM C215 presupposes the use of sensors coupled to the samples (accelerometers). On the other hand, ASTM E1876 describes both the use of contact sensors and sensors with no need of coupling (this second option is the most recommended because there is no interference of the sensor with the vibration of the sample);

— Equations for the elastic moduli calculation:

Table 2 describes the equations relative to the calculation of respective elastic moduli according to the vibration mode.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexural</td>
<td>$E = 1.6067 \cdot \frac{L^3 \cdot m \cdot f_f^2 \cdot T}{d^4}$</td>
<td>$E = 1.6067 \cdot \frac{L^3 \cdot m \cdot f_f^2 \cdot T_1}{d^4}$</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>$E = 5.093 \cdot \frac{L \cdot m \cdot f_l^2}{d^2}$</td>
<td>$E = 5.093 \cdot \frac{L \cdot m \cdot f_l^2}{d^2 \cdot K}$</td>
</tr>
<tr>
<td>Torsional</td>
<td>$G = 16 \cdot \frac{L \cdot m \cdot f_t^2 \cdot R_{C215}}{\pi \cdot d^2}$</td>
<td>$G = 16 \cdot \frac{L \cdot m \cdot f_t^2 \cdot R_{E1876}}{\pi \cdot d^2}$</td>
</tr>
</tbody>
</table>

Where $E$ is the modulus of elasticity (in Pascal); $L$ is the length of the sample (given in millimeters); $d$, the diameter (in millimeters); $m$, the mass (in grams); $f_f$, the fundamental flexural frequency (in Hertz); $f_l$, the fundamental longitudinal frequency (in Pascal); $f_t$, the fundamental torsional frequency (in Hertz); $T$, the correction factor of the flexural mode (ASTM C215); $T_1$, the correction factor of the flexural mode (ASTM E1876); $R_{C215}$, the correction factor of the torsional mode (ASTM C215); and finally $R_{E1876}$, correction factor of the torsional mode (ASTM E1876).

As it can be observed, the differences between the equations are their respective correction factors (T factor, for the bending test, K factor for longitudinal test, and R, for torsion test). Whilst ASTM C215 uses as a reference the calculations performed by Pickett [38], ASTM E1876 uses as a reference the calculations from Spinner and Tefft [39]. The difference between these references is the fact that the last refined the
calculations performed by the first, increasing a correction factor (in the case of the equation for longitudinal mode), or adjusting the present correction factors. As a result, it is recommended to use the equations described by the ASTM E1876 standard [1] for the calculation of the elastic moduli. But it is worth highlighting that the percentage difference between the values obtained using both standards will depend on the Poisson’s ratio and on the aspect ratio of the sample, considering that for concrete samples with aspect ratio of d/L = 0.5 and Poisson’s ratio equal to 0.20, the percentage difference may get up to 5% in the elastic modulus measured by the longitudinal vibration test (generally speaking, the greatest differences between the obtained values are associated with this vibration mode).
6. Estimation of the static modulus of elasticity of a concrete employing the proposed methodology

The goal of this chapter is to illustrate the application of the methodology for the prediction of the chord modulus (ASTM C469) of a concrete sample using the Sonelastic® solutions and the Impulse Excitation Technique described in Chapter 4. Next, find the step-by-step procedure:

**Step 1 – Determining the geometry and dimensions of the sample:**

As suggested, for this study a concrete cylinder with 100 mm of diameter and 200 mm of length was used.

**Step 2 – Preparing, measuring and weighing the sample:**

Aiming to increase the dimensional precision, the faces of the sample were grinded. Table 3 describes the mass and dimensions, as well as the apparent density (ratio between the mass and volume of the sample).

<table>
<thead>
<tr>
<th>Table 3 – Dimensions, mass and apparent density of the concrete sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
</tr>
<tr>
<td>3725.33 ± 0.01</td>
</tr>
</tbody>
</table>

The uncertainty of the density was calculated from the mass and dimensions uncertainties. Next, find the uncertainty calculation of this property for the cylinder used in this example:

\[
\Delta \rho = \rho \cdot \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta L}{L}\right)^2 + \left(2 \cdot \frac{\Delta D}{D}\right)^2} = 2.354 \cdot \sqrt{\left(\frac{0.01}{3725.33}\right)^2 + \left(\frac{1.0}{199.5}\right)^2 + \left(2 \cdot \frac{0.65}{100.49}\right)^2} = 0.033
\]

**Step 3 – Characterizing the dynamic modulus of elasticity using the IET:**

For the application of the Impulse Excitation Technique, the Sonelastic® equipment was employed according to following configuration: 3.0 Sonelastic® Software, adjustable support for bars and cylinders SA-BC, directional microphone CA-DP and manual impulse device (Figure 20). As described in Chapter 5, there are two vibration modes that may be used to determine the dynamic, longitudinal and flexural modulus of elasticity. Figure 18c illustrates the positioning of the sample and the points of excitation and signal acquisition for the longitudinal mode. Figure 18a and Figure 20 illustrates the same positioning for the flexural mode.
After applying the boundary conditions described, the mechanical impulse was applied to the sample and, immediately after, the acoustic response was acquired. The Sonelastic® software processed the acoustic response and through its interface it was possible to obtain the modulus of elasticity for the excited vibration mode.

Table 4 – Modulus of elasticity as a function of the vibration mode:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Longitudinal</th>
<th>Flexural</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (GPa)</td>
<td>37.62 ± 0.64</td>
<td>36.39 ± 1.28</td>
</tr>
</tbody>
</table>

Step 4 – Apply the Popovics’ model for the estimating the static elastic modulus of elasticity:

The next step consisted in applying the Popovics model for the estimation of the chord elastic modulus (ASTM C469). For the model application, it was used the dynamic elastic modulus ($E_d$) obtained from the longitudinal vibration mode because of its correspondence with the compression test (item 5.3). Next, find the calculation for the estimation of the chord elastic modulus:

$$E_c = 0.107 \cdot E_d^{1.4} \cdot \rho^{-1} = 0.107 \cdot (37.62 \cdot 10^9)^{1.4} \cdot 2354^{-1} = 29.05 \, \text{GPa}$$

For the determination of the uncertainty, the following expression was used:

$$\Delta E_c = E_c \cdot \sqrt{(1.4 \cdot \frac{\Delta E_d}{E_d})^2 + \left(\frac{\Delta \rho}{\rho}\right)^2} = 29.05 \cdot \sqrt{(1.4 \cdot \frac{0.64}{37.62})^2 + \left(\frac{0.033}{2.354}\right)^2} = 0.80 \, \text{GPa}$$

Therefore, the static modulus of elasticity of the analyzed concrete corresponds to:

$$E_c = 29.05 \pm 0.80 \, \text{GPa}$$
**Step 5 – Make a report including the results and main parameters:**

The last step consisted in filling the report described in item 4.3 (pg 19, Figure 15), which should favor the inclusion of the main information regarding the material, type of test, and applied model for the estimation of the static modulus of elasticity.

### Report of static modulus of elasticity estimation (chordal) by the dynamic elastic modulus obtained with Impulse Excitation Technique

**General information**  
Responsible: Eng. Lucas Otani  
Place: ATCP Iq  
Date: 30/07/2015  
Time: 14h00

**Material and test information**

- **Name / Part number:** CP-01  
- **Mix:** 1 : 1.30 : 1.73  
- **Water / cement ratio:** 0.45  
- **Type of aggregate:** Gravel  
- **Cure conditions:** Wet chamber (23 °C)  
- **Age:** 28 days  
- **Sample geometry:** Cilindro  
- **Dimensions:** L = 199.5 ± 1.0 mm and D = 100.49 ± 0.65 mm  
- **Mass:** 3725.33 ± 0.01 g  
- **Density:** 2.354 ± 0.033 g/cm³

* Include the respective uncertainties.

**Calculation of apparent density uncertainty:**  
* Δρ = ρ ± \[ \sqrt{\left( \frac{\partial \rho}{\partial \text{water content}} \right)^2 + \left( \frac{\partial \rho}{\partial \text{cement content}} \right)^2} \]

**Information on the non-destructive test (Impulse Excitation Technique)**

- **Temperature and humidity:** 26°C / 40%  
- **Standard used:** ASTM E1876-09  
- **Vibration mode:** Longitudinal  
- **Equipment used:** Sonelastic

**Information on the model used to estimate the static modulus of elasticity**

- **Model used:** Popovics  
  \[ E_c = 0.107 \cdot E_d^{1.4} \cdot \rho^{-1} \]

* We suggest the model proposed by Popovics:

  \[ E_c = 0.107 \cdot E_d^{1.4} \cdot \rho^{-1} \]

The static modulus of elasticity estimated by the Popovics model \( E_c \) corresponds to the chordal modulus obtained between two points of the stress-strain graph at the first at a deformation level of 5.00E-5 and the second at 40% of the compressive strength.

**Results:**

- **Dynamic modulus of elasticity \( E_d \):** 37.62 ± 0.64 GPa

- **Estimated static modulus of elasticity \( E_c \):** 29.05 ± 0.80 GPa

* From the model proposed by Popovics, uncertainty can be obtained by:

  \[ \Delta E_c = E_c \cdot \sqrt{\left(1 + \frac{\Delta E_d}{E_d}\right)^2 + \left( \frac{\Delta \rho}{\rho} \right)^2} \]

**Figure 21** – Results report with the main parameters and the methodology used.
7. Suggestion for future works

The application of the Impulse Excitation Technique has been rising because engineers and scientists have been increasingly seeking for more precise, easier to use and cost effective techniques. In addition, by applying this non-destructive technique, it is possible to characterize the same sample many times and for more than one property, reducing the results dispersion and number of samples.

There is a great number of potential studies in which the application of the Impulse Excitation Technique and the Sonelastic® equipment could be used for the characterization of concrete samples. Among them, it is possible to mention the followings:

1. Validation of the Popovics’ model (Topic 4.2) for different types of commercial concretes, considering the formulations employed and locally available raw-materials;

2. Development and validation of correlation models between the dynamic and the static elastic moduli for special concretes which have incorporated fibers, scraps, special additives, among other;

3. Development and validation of correlation models for all types of materials used in civil construction, such as mortar, rocks, clay, pottery, coatings, among others;

4. Verification of the possibility of using the Impulse Excitation Technique for the quality control of concretes in the building site;

5. Verification of the correlation between the dynamic modulus of elasticity and the compressive strength (f_{ck}) of the concrete and, through controlled conditions, getting to the stage where it is possible to precisely estimate this property only through the evaluation of the dynamic elastic modulus. Nowadays, this correlation is performed in the opposite sense when it comes to the static modulus of elasticity.
8. Final statements

The current technical-informative white paper proposed a reflection about the characterization and estimation of the static modulus of elasticity of concretes, as well as it presented an alternative and non-destructive methodology for the estimation of the static modulus of elasticity that has great growing potential considering the next years.

In addition, the main aspects regarding the employment of the Impulse Excitation Technique for the characterization of the dynamic elastic moduli of concretes were also described.

Finally, it was described some possibilities for future works which relevance will increase more and more as a result of the search for more precise and effective routes for measuring these properties and for the quality control of concretes.
9. References


Appendix – Procedure for coupling tabs to a cylinder for characterizing the dynamic shear modulus

As described in Chapter 5, for the characterization of the shear modulus, it is necessary to apply the torsional vibration mode (Figure 18b). Exciting this mode in cylindrical samples is only possible by coupling two small blocks of metal (15 mm x 10 mm x 5 mm) to the sides of the sample, as Figure 22 illustrates. These blocks (tabs) must be well fixed to the sample since the excitation and signal acquisition will be taking place right at these points. For such procedure, it is suggested the use of the epoxy glue, as well as a hole should be made at the center of the metal block to increase the contact area of the surface for the glue. See the representation of how the tabs should be placed in Figure 22.

![Figure 22 - Representation of the tabs placement on the cylindrical sample.](image)

To apply the torsional vibration mode, the excitation and signal acquisition must be positioned in a way that they are directed to the tabs coupled to the sample (Figure 18b), as Figure 23 illustrates.

![Figure 23 - Excitation and signal acquisition positioning for the torsional vibration mode.](image)